**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answer:**

**X = 600**

**Meam = 45+10 = 55**

**Std. Deviation = 8**

**from scipy import stats**

**round(1-stats.norm.cdf(60,loc=55,scale=8),5)**

**0.26599**

**Option B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**False, since mean = 38 and standard deviation = 6, that means most of the ages are lying between 32 and 44.**

**Z score for 44**

**from scipy import stats**

**round(1-stats.norm.cdf(44,loc=38,scale=6),5)**

**0.15866 = 63 people out of 400**

**Z score between 38 and 44**

**from scipy import stats**

**stats.norm.cdf(44,loc=38,scale=6)-stats.norm.cdf(38,loc=38,scale=6)**

**0.3413447460685429 = 137 people out of 400**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True**

**from scipy import stats**

**stats.norm.cdf(30,loc=38,scale=6)**

**0.09121121972586788**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer:**

**As we know that if X <" N(µ1, Ã 1^2 ), and Y <" N(µ2, Ã 2^2 ) are two independent random variables then X + Y <" N(µ1 + µ2, Ã 1^2 + Ã 2^2 ) , and X " Y <" N(µ1 " µ2, Ã 1^2 + Ã 2^2 ) .**

**Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(aµ1 + bµ2, a^2Ã 1^2 + b^2Ã 2^2 ).**

**Therefore in the question**

**2X1~ N(2 u,4 Ã ^2) and**

**X1+X2 ~ N(µ + µ, Ã ^2 + Ã ^2 ) ~ N(2 u, 2Ã ^2 )**

**2X1-(X1+X2) = N( 4µ,6 Ã ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer:**

**Since it is 99% we need to add 0.5% on either side, i.e 0.005 = 0.99+0.005 = 0.995**

**Now Z value of 0.005 = from scipy import stats**

**stats.norm.ppf(0.005)**

**-2.575829303548901**

**Now Z value of 0.995 = from scipy import stats**

**stats.norm.ppf(0.995)**

**2.5758293035489004**

**Z = (x-Mean)/std.D , x = std.D\*Z + Mean , x = 20\*Z + 100**

**A= (20 \* (-2.57)) + 100 = 48.5**

**B = (20\* 2.57) + 100 = 151.5**

**option D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**import numpy as np**

**from scipy import stats**

**from scipy.stats import norm**

**Mean = 5+7**

**print('Mean Profit is Rs', Mean\*45,'Million')**

**SD = np.sqrt((9)+(16))**

**print('Standard Deviation is Rs', SD\*45, 'Million')**

**print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')**

**Mean Profit is Rs 540 Million**

**Standard Deviation is Rs 225.0 Million**

**Range is Rs (99.00810347848784, 980.9918965215122) in Millions**

1. Specify the 5th percentile of profit (in Rupees) for the company

**From Z table we know that 5th percentile value is -1.645**

**X = std.D\*Z + Mean = import numpy as np**

**X= 540+(-1.645)\*(225)**

**print('5th percentile of profit (in Million Rupees) is',np.round(X,))**

**5th percentile of profit (in Million Rupees) is 170.0**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Making loss i.e X<0**

**Division 1**

**stats.norm.cdf(0,5,3)**

**0.0477903522728147**

**Division 2**

**stats.norm.cdf(0,7,4)**

**0.040059156863817086**

**Division 2 will face more loss**